Charge screening and mass spectrum

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(Received 19 May 1975)

A general connection between the existence of charge sectors and the mass spectrum is established in Abelian gauge theories in a space-time of dimension larger than two. The exceptional role of two-dimensional space-time is related to Coleman's "quantum soliton."

Currently a great deal of interest is being devoted to solutions of nonlinear field theories in their classical,[1,7] semiclassical[2,3] and fully quantized[4-6] aspects. A typical feature of those solutions is the appearance of sectors associated with identically conserved currents, i.e., of charges that are not zero only because of the existence of long-range states. In this paper we show that in a space-time of dimension larger than two identically conserved currents lead to zero charge (no charge sectors) unless there are zero-mass states in the theory. This result generalizes what has been known as the charge-screening effect[8] in connection with the Higgs[9] and the Schwinger[10] models.

The failure of the general proof in two dimensions allows for the existence of sectors associated with identically conserved currents in theories without zero-mass states as in the Goldstone-Jackiw[3] and Coleman[4] recent proposals. The peculiar feature of two-dimensional space-time arises from the fact that in this case any conserved current can be derived from a local potential, as long as there is a mass gap, being in this way identically conserved.

Consider an identically conserved current

$$j^{\mu}=\partial_{\nu}F^{\mu\nu}\ , \tag{1}$$

where in two dimensions we have

$$F^{\mu\nu}=\epsilon^{\mu\nu}\phi\ . \tag{2}$$

We wish to know under which conditions will there be sectors corresponding to nonzero values of the charge formally given by

$$Q=\int j^0 d\underline{x}. \tag{3}$$

From Gauss's law we know[11] that the charge sectors, if they exist, will correspond to long-range states, i.e., states that cannot be obtained by applying local charge-raising operators to the vacuum. What we can show is that the existence of charge sectors is incompatible with the assumption of a mass gap in the theory. In other words, a long-range charged states imply the existence of massless photons. Conversely, whenever the "photons" acquire a mass (via a Higgs mechanism or any other) the charge is screened.

The proof goes as follows: Suppose there were charged states in a theory with a mass gap and which is asymptotically complete where (1) holds. Consider a charged one-particle state $|p\rangle$. For simplicity we take spin-0 states, the generalization to higher spin being straightforward. With

$$\langle p|j^{\mu}(0)|p'\rangle=(p+p')^{\mu}G(t)\ ,\quad t=(p-p')^2 \tag{4}$$

$$\langle p|F^{\mu\nu}(0)|p'\rangle=[(p-p')^{\mu}(p+p')^{\nu}-(p+p')^{\mu}(p-p')^{\nu}]F(t)\ , \tag{5}$$

we get from (1)

$$F(t)=i\,\frac{G(t)}{t}\ . \tag{6}$$

The existence of a nonzero charge implies therefore that the form factor of $F^{\mu\nu}$ develops a pole at the origin. Although this could be taken as an indication for the presence of zero-mass particles in the theory, we should remember that the usual analyticity structure of the form factors depends on the locality of the interpolating charge field (with respect to $F_{\mu\nu}$), an assumption one cannot make here.

A more detailed investigation of the connection between the pole in (6) and the mass spectrum of the theory is required: We show that in space-time of dimension larger than two the pole violates the locality of the $F^{\mu\nu}$ field if there is a mass gap.

To see that let us take the commutator

$$\langle \underline{p}=0|[F^{0i}(\underline{x},0),j^i(g)]|\underline{p}=0\rangle=C(\underline{x})\ , \tag{7}$$

where there is *no* summation over i, and in a more rigorous treatment we would have taken normalized states with momentum centered around zero instead of the improper states $|\underline{p}=0\rangle$. In an asymptotically complete theory without zero-mass particles there is necessarily a mass gap between the one-particle hyperboloid and the continuum of states in the charge-one sector. We choose our test function g to be of the Schwartz class with its Fourier transform having the following properties:

$$\tilde{g}(p)=0\,, \quad |p_0|>\delta\,, \quad \delta<\text{mass gap}\,, \tag{8a}$$

$$\tilde{g}(p)=\tilde{g}(-p)\,, \tag{8b}$$

$$\tilde{g}(0)=1\,. \tag{8c}$$

Locality of the $F^{\mu\nu}$ implies that $C(\underline{x})$ goes to zero faster than any inverse power of $|\underline{x}|$, since g is of the Schwartz class

$$|C(\underline{x})|<\frac{A_k}{|\underline{x}|^k} \text{ for any } k\,. \tag{9}$$

Because of support property (8a) of $\tilde{g}$ only the one-particle intermediate states contribute to the commutator (7) so that

$$C(\underline{x})=i\,4m\int d\underline{p}\,\frac{e^{i\underline{p}\cdot\underline{x}}}{(\underline{p}^2+m^2)^{1/2}}\frac{G^2(t)}{t}\times\tilde{g}(\underline{p},(\underline{p}^2+m^2)^{1/2}-m)p^{i2}\,, \tag{10}$$

where m is the mass of the charged particle.

The asymptotic behavior of (10) for large $|\underline{x}|$ is given by

$$C(\underline{x})\sim iG^2(0)\int d\underline{p}\,e^{i\underline{p}\cdot\underline{x}}\frac{p^{i2}}{\underline{p}^2}=-iG^2(0)\frac{2^n\pi^{n/2}\Gamma(n/2)}{|\underline{x}|^{n+2}}(nx^{i2}-|\underline{x}|^2)\,, \tag{11}$$

where $n+1$ is the space-time dimension. It is clear that for any space-time dimension larger than two (11) is only compatible with (9) if $G(0)=0$, that is, if there are no charge sectors.

A few remarks are now in order. Our proof is geared to Abelian gauge theories where $F^{\mu\nu}$ can be safely taken as a local field. In the non-Abelian case the $F_a^{\mu\nu}$ carry charge (color) themselves and will in general be nonlocal in the physical Hilbert space. However, even in a non-Abelian gauge theory some conclusions can be drawn: A quantum analog of 't Hooft's magnetic monopole solution[1] will lead to sectors labeled by a magnetic charge associated to the current

$$k^\mu=\partial_\nu\epsilon^{\mu\nu\sigma\rho}F_a^{\sigma\rho}\phi^a\,,$$

where a is the color label and ϕ the Higgs field. Since it is color neutral $\epsilon^{\mu\nu\sigma\rho}F_a^{\sigma\rho}\phi^a$ should be a local field, and therefore the existence of magnetic sectors implies necessarily the presence of massless "photons."

We believe that by exploiting the locality of color-neutral composite fields a more general connection between the existence of color sectors and the mass spectrum of theory should follow. This would be of some relevance for gauge theories of "quark confinement."[12]

The reason we could not obtain any result in two dimensions is clear: In this case in any massive theory with a conserved current we can always introduce a local (with respect to itself) field

$$-\phi(x)=\frac{2\pi}{\beta}\int_{x_1}^{\infty}j^0(x_0,x_1')dx_1'\,, \tag{12}$$

which acts as a potential for the current,[13] i.e., with (2), Eq. (1) is satisfied. The existence of (12) as an operator-valued distribution follows by using arguments parallel to those which establish the existence of charges in local theories with a mass gap.[14] The crucial point is that $\langle 0|\phi(f)\phi(f)|0\rangle$ is finite, what follows immediately from the current-current two-point function.

If the massive theory has a scale-invariant high-energy asymptote the Schwinger term will be finite (as in the Thirring model), and therefore by an appropriate choice of β in (12), $\phi(x)$ will satisfy canonical equal-time commutation relations. In this case, assuming the charge is the only internal degree of freedom in the theory (the current is irreducible in every charge sector), we expect

$$\Box\phi=F(\phi)\,, \tag{13}$$

with $F(\phi)$ defining a superrenormalizable interaction. If, furthermore, there exists a charge-raising field in the theory, local with respect to j^μ (although of course not with respect to ϕ) implying a conventional additive charge sector structure,[15] the validity of (13) in any charge sector leads to

$$F\left(\phi+\frac{2\pi}{\beta}\right)=F(\phi)\,. \tag{14}$$

In this way we are heuristically led to Coleman's relation between the sine-Gordon and massive Thirring models[4] and its trivial generalizations: For the superrenormalizability of the theory requiring the highest Fourier component of (14) to have an angular frequency less than $\sqrt{8\pi}$ we may for $\beta<\sqrt{8\pi}/n$ introduce besides the basic $\sin\beta\phi$ interaction some higher harmonics corresponding to the existence of asymptotically vanishing perturbations other than the mass term, such as, for instance,

$$\lim_{\epsilon\to 0}\epsilon^{-\beta^2/4\pi}[:\bar\psi(x+\epsilon)\psi(x+\epsilon)::\bar\psi(x)\psi(x): -\langle:\bar\psi(x+\epsilon)\psi(x+\epsilon)::\bar\psi(x)\psi(x):\rangle]\,. \tag{15}$$

It should be clear from the preceding remarks the exceptional role played by two-dimensional field theories in the construction of charge sectors associated with identically conserved currents in theories with a mass gap. One also realizes that in the Schwinger model[10] the screening occurs not because of Gauss's law [Eq. (1)], which is always trivial in two dimensions, but through the coupling of the electric potential to the "electron" field. The absence of such a nonlocal coupling in the massive Thirring model allows for the locality of the Thirring field with respect to itself as well as for the existence of charge sectors.

The author wishes to thank the theoretical group of New York University for their very kind hospitality.

*Work supported in part by the National Science Foundation, Grant No. GU-3186.

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